Sky coverage in layer-oriented adaptive optics

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ABSTRACT

In layer–oriented adaptive optics, multiconjugation is performed in a much more efficient way than conventional wavefront sensing. This improved efficiency is impressive for high altitude layers and moderate for ground ones. On the other hand high altitude layers can be covered with only a limited field of view (where one can search for natural guide stars) while for ground layers the usable field of view is limited essentially by practical reasons. We introduce the further concept of multiple field of view layer oriented where a combination of sampling and covered field leads easily to sky coverages that nearly approach the whole sky with the usage of solely natural guide stars for 8m class telescopes.

Keywords: multi-conjugation – multiple reference wavefront sensor

1. INTRODUCTION

Multi Conjugate Adaptive Optics (MCAO) has been introduced by Beckers\textsuperscript{1} as a way to overcome Field of View (FoV) limitations, typical of classical Adaptive Optics\textsuperscript{2} (AO). In MCAO more than one Deformable Mirror (DM) is conjugated to a specific height in the turbulent atmosphere in a way that, up to a certain extent, one can cancel out the atmospheric turbulence in a three–dimensional way, although with a strong discretization along the range direction. Even if MCAO essentially refers to the way the DMs introduce the correction in the optical path, a key role is played by the way these DMs are driven or, in other words, by the WaveFront Sensor (WFS) unit(s). Different schemes have been proposed, ranging from open loop tomography\textsuperscript{3,4} to closed loop approaches, namely the global reconstructor\textsuperscript{5} and the layer–oriented one.\textsuperscript{6,7}

In the layer-oriented approach several reference stars are simultaneously sensed by a single WFS with one or more detectors that can be optically conjugated to a specific altitude. Coupling each detector with a specific DM, conjugated to the same layer, allows for extremely efficient closed loop operations. Of course a given DM corrects not only the conjugated layer, but a whole slab of atmosphere: each non-conjugated layer is corrected up to a spatial frequency which is inversely proportional to the distance from the conjugation plane. Layer–oriented MCAO has been studied with regards to stability and it has been found essentially equivalent to the global reconstructor MCAO approach in a set of interesting conditions.\textsuperscript{8} Moreover it is easy to see that it is nearly equivalent to the optimum achievable correction, at least in a linear sense.\textsuperscript{9,10}

Each independent AO loop forming a layer–oriented system corrects a specific volume of atmosphere and therefore the spatial and temporal sampling can be finely tuned to match the properties of the turbulence in that specific volume. In this way, for instance, the detector conjugated to the high altitude layer could be used in a sampling mode embracing a local $r_0$ that can be significantly larger than the ground one and, on the opposite, wind speeds on the ground being significantly lower than the high altitude ones, integration times for the...
ground layer conjugated detector can be significantly larger. Since the complexity of the layer-oriented system essentially scales with the number of DMs rather than with the number of sensed references, it is possible to sense simultaneously a huge number of stars, optically co-adding the light of even very faint sources, which would be otherwise useless. Furthermore the layer-oriented approach fits well with any pupil-plane WFS, although it has been shown that this is not necessarily a strict requirement\[11\]; a suitable choice is the Pyramid WFS,\[12\] which is known to exhibit significant gain in terms of limiting magnitude with respect to the Shack—Hartmann one.\[13,14\]

All these concepts combined together lead to the easy speculation that, provided all these conditions are successfully met, good sky coverages could be obtained with the usage of solely NGSs, solving the tip–tilt indetermination problem\[15,16\] and of course all the technical issues related to the use of a significant number of artificial reference beacons. In this paper we describe a new concept, mostly based upon the layer-oriented approach, and we show that it allows for relevant sky coverages, of the order of one third of the sky at the Galactic Poles and approaching the full coverage on the Galactic equator and at moderate Galactic latitudes, even without using any of the considerations pointed out above.

2. THE MULTIPLE FIELD-OF-VIEW CONCEPT

In layer-oriented AO the concept of limiting magnitude, common in conventional AO, and to a certain extent applicable also to classical MCAO, is lost in favour of the concept of equivalent photon density at a given layer. Such a photon density is obtained by the concurrent fluxes of several stars. The achievable correction depends upon the light coming from different references, in a way that these co–add to produce a Signal to Noise Ratio (while sensing the atmospheric turbulence) higher than the one achievable by simple AO. In layer-oriented this is obtained optically, in a way that easily allows for the collection of light coming from several, very faint, sources.

There are, however, some limitations on the achievable gain in layer-oriented, related to the existence of an upper limit on the FoV in order to increase the photon density for a given altitude layer. In fact, increasing the FoV beyond some limit angle \(\theta_L\), the pupil footprints projected onto a high altitude layer are displaced in a way that the starlight is spread onto a larger and larger area, with no net density improvement. Moreover, it has to be recalled that increasing the FoV, other than having a technical cost, has to be paid also in terms of correction efficiency in the portion of atmosphere far away from the locations where the DMs are conjugated.

We want now to point out that the spatio-temporal sampling gain offered by layer oriented may be huge for the high altitude portions of the atmosphere, but it may be small on the ground layer. Roughly speaking, an increase of the local \(r_0\) translates into a photon collection increase that scales with the third power of such increase. In fact the usable portion of the pupil scales as \(r_0^2\), while the integration time scales linearly, leading to the cubic power scaling law. The local \(r_0\) of the ground layer is quite close to the \(r_0\) of the overall atmosphere and this explains the limited gain achievable in this case.

The previous discussion can be summarized by the following key statements:

- the significative gain for a correction in the layers close to the ground can be achieved essentially by the adoption of a large FoV where enough photons can be collected efficiently from different directions on the sky;
- on the opposite side, the gain in the high altitude layer due to the FoV extension is limited, but here one can exploit the spatio-temporal gain offered by the layer-oriented approach.

The more straightforward approach (but not the only one) to take into consideration the key points mentioned above is to use different covered Field of View for the different detectors. Basically, in its simpler form, the Multiple-FoV layer-oriented MCAO concept can be described as follows:

- A nearly-ground layer with a large FoV; this allows for the compensation of the lowest portion of the atmosphere. There will be a substantial gain in photon density, because of the large FoV. On the other hand the collected starlight has to be sampled essentially with the same spatio-temporal sampling of the whole atmosphere column;
Figure 1. A MCAO system with two DMs is considered. One DM is conjugated to the ground, or very close to this, while the second is conjugated at a certain altitude. The correction for the first DM is driven by using reference stars embracing a FoV of, in the described example, 6 arcmin, while the second DM is driven by a second set of reference stars embracing a smaller FoV, in the drawing of 2 arcmin. Layers not exactly conjugated to the DMs will be corrected only at spatial frequencies lower than the ones defined by the footprints of the beams embracing the selected stars. In this way the correction performed by the DM with a larger FoV will degrade more rapidly with the distance of the layer from the altitude where the DM is conjugated. Three layers are shown with the footprint of the beam of the closer DM, just to simplify the layout.

- A high-altitude layer with a FoV of the order of the limiting angle $\theta_{\gamma}$, where the gain is accomplished through a more efficient spatio-temporal sampling.

In the following we will assume an inner FoV of $1'$ in radius, representing the science field, and an annular FoV with inner and outer radius of $1'$ and $3'$ respectively. These figures are just indicative and no optimization has been attempted. It is noticeable that only the small portion of the FoV is fully corrected, at least in a MCAO sense. The annular region concerning the larger FoV is corrected only for ground turbulence and, although both compensations will work in a closed loop fashion, the achieved correction will be limited on the annular and larger FoV. As the annular FoV is working in closed loop with no correction of the high altitude portion of the atmospheric turbulence, the correction of the ground layer will be, moreover, perturbed by the high altitude layers. Strictly speaking the introduced compensation should correct (at least in the high SNR regime) the ground turbulence and should apparently replace it with superimposed replicas of the high altitude layers. That means that, after the ground turbulence correction, some very low spatial frequency residual will still be introduced. Although the physical origin of these residuals are in the high altitude layers, these will appear optically, inside the MCAO system, as coming from the ground. One can say, in other words, that the effect of the layer-oriented closed loop conjugated on the large FoV on the ground will replace the ground layer with an hampered version of the high altitude turbulence.

From the practical point of view, the system may be realized with two DMs and three detectors (Figs. 1, 2). The DMs are conjugated with the ground and the high altitude layer. The first detector is conjugated to the ground layer, looking at a ring projected on the sky with inner radius $1'$ and outer radius $3'$, with spatial and temporal sampling typical of the lowest part of the atmosphere at the wavefront sensing wavelength, assumed to be $0.8\mu m$. The second detector is again conjugated to the ground, looking at a circular FoV of $1'$ radius with spatial and temporal sampling typical of the highest part of the atmosphere, suitably adjusted in order to take into account the smoothing effect of the wavefronts, due to the superposition of the star footprints in the annular FoV. The third detector is conjugated to the high altitude layer, looking at the same circular FoV of $1'$ radius.
Figure 2. Schematic layout of the Multiple FoV system, with two DMs and three detectors. The signal of the two detectors conjugated to the ground is sent to the same DM.

Figure 3. An overall sketch of a possible opto-mechanical configuration. The inner, 2 arcmin wide, FoV is folded by a mirror located close to the focal plane of the telescope and sent, through a 1:1 optical relay, to a magnetic positioner of reflective pyramids, that are to be accommodated by a robotic arm to sense the 20 brightest stars. These pyramids reflect the splitted light into an objective and, through a beam splitter, the light is fed to two CCDs conjugated to the ground and high altitude layer respectively. The annulus with external diameter of 6 arcmin interest a number of refractive pyramids set up in place by a polar robotic positioner and fed, through a dedicated objective, to a CCD conjugated to the ground layer. Fiber tapers are part of the optical design allowing to a reasonable size for the pupils to be reimaged onto the CCDs, allowing the usage of commercially available ones.
of the previous detector, with spatial and temporal sampling typical of the highest part of the atmosphere. The signal of the first two detectors, conjugated to the ground, is sent to the same DM.

We have also conceived a possible optomechanical layout of the Multiple FoV system, shown in Fig. 3. Assuming a typical 8m Ritchey-Chretien telescope, we have first corrected the field curvature by a simple field flattener, ensuring diffraction-limited imaging performance over a FoV of 6'. The beams produced by the pyramids placed on the two focal planes are focused onto the corresponding detectors by means of two F/2.5 optical relays; a fiber taper shrinks the image size by a further factor of 5, in order to match the size of the pupil image to the detector. The imaging performance is such that 80% of the total energy is ensquared in a region smaller than 1/10 of the equivalent $r_0$ size: this ensures that the spatial resolution of the relays is much better than the sub-aperture size for the layer oriented wavefront sensing.

3. SKY COVERAGE CALCULATION

The sky coverage may be defined as the fraction of the sky where the average Strehl Ratio (SR) across the FoV is at least 50% of the value achievable in an ideal noise-free case. In general the achievable SR depends on the number and conjugation range of the DMs (Fig. 4) and on the accuracy with which the turbulence is measured. Of course the latter figure depends on the photon density on the turbulent layers. In other words, for a given DMs configuration and turbulence profile, the SR depends on the availability of a sufficient number of reference stars in the FoV.

![Figure 4](image-url)

**Figure 4.** A plot of $C_n^2$ vs. both altitude $h$ and spatial frequency $k$ with the portion deleted from a couple of DMs. These are represented by the empty bands denoting a couple of DMs conjugated respectively at 800m and $\approx 8$km. The turbulence at such altitudes is completely removed while the turbulence at different altitudes is compensated only up to a given spatial frequency, because of the smoothing effect associated to the covered FoVs. The DM conjugated to the ground adopts a larger FoV and hence the degradation of the correction is faster than for the other DM, characterized by a smaller FoV. In this picture the DMs are driven by a Multiple-FoV layer oriented AO with respectively 6 and 2 arcmin of FoV. Finally on the bottom graph the integral along $h$ of the turbulence is shown exhibiting a spectrum with only high spatial frequencies residuals.

In our calculations we have considered typical turbulence profiles. We have adopted a Multiple-FoV system with two DMs, conjugated at 0 and 5Km, and three detectors, each with the proper spatial and temporal sampling. Concerning the noise propagation in the wavefront estimation, we have followed the approach described in Rigaut & Gendron and we have implemented the noise propagation coefficients of the Shack-Hartmann WFS. We have found that, in order to have the desired level of correction, an integrated magnitude $R \approx 16$ is necessary.
The stars distribution in the sky has been modeled according to Bahcall & Soneira\(^{18}\) in the R band: for each field of interest, at most the 20 brightest available stars have been considered. The plots of the integrated R magnitude are shown in Fig. 5, whereas the sky coverage is reported in Tab. 1. The main result is that, in regions close to the Galactic plane, the sky coverage is \(\approx 90\%\) and decreases to \(\approx 30\%\) at the Galactic Pole, a rather large value indeed, considering the much lower density of stars and that no optimization of the system has been attempted.

![Figure 5](image-url)

**Figure 5.** Left: equivalent sky coverage, normalized to unity, for the annular 6' FoV, plotted as a function of the integrated R magnitude for different choices of the Galactic coordinates \((l, b)\). Only the 20 brightest available stars have been considered. Right: sky coverage for the central 2' annular FoV.

<table>
<thead>
<tr>
<th>Galactic latitude</th>
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<th>central FoV</th>
<th>combined probability</th>
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**Table 1.** Sky coverage for different Galactic latitudes.

4. CONCLUSIONS

The Multiple-FoV layer-oriented approach allows for relevant sky coverage, of the order of one third of the sky at the Galactic Poles and approaching full coverage on the Galactic Equator and at moderate Galactic latitudes.

The Multiple-FoV approach has been described in its simplest form. Of course further modifications are possible. In case there is a strong requirement in term of Strehl Ratio (without FoV reductions) or in case the atmospheric characteristics of a telescope site are such that there is a strong part of the turbulence located far away from the layers where the DMs are conjugated, the introduction of further DMs may be required. There is an almost ‘natural’ way to generalise the Multiple FoV Layer Oriented concept by considering different Field of Views for each specific DM. In the simplest case of three DMs (Fig. 6), one should consider three different FoVs for the three DMs conjugated to the three different altitudes and relative detectors.

Concerning possible extensions to Extremely Large Telescopes (ELTs), the major problem, at least at a first glance, is represented by the size of the optical relays, if these are simply scaled up starting from the design presented here. An alternative design, optimized for a 100m class telescope, has been proposed\(^{19}\), characterized by a long optical train and a reasonable size of the various lenses.
In our evaluation of the sky coverage we have made no attempt to optimize a number of parameters that might be used for the fine tuning of the system, namely FoV size, DMs conjugation range and number of stars. Furthermore we have not considered the gain achievable with the pyramid WFS, which fits naturally the proposed approach and might lead to noticeable improvements.

Figure 6. Generalizing the multiple-FoV concepts to more than two DMs, one can conceive different FoVs for each specific DM. One should also note that, because of the different smoothing of the layers distant from the DM conjugation, the covered volumes are likely to be roughly inversely proportional to the different FoVs: a smaller range for the larger FoV on the lowest DM, and progressively larger volumes for smaller FoVs at higher distances from the entrance pupil.

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REFERENCES